Family Replicated Gauge Group Models*

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Family Replicated Gauge Group models of the type $SU(n)^N \times SU(m)^N$, $(SMG)^3$ and $(SMG \times U(1)_{B-L})^3$ are reviewed, where $SMG = SU(3)_c \times SU(2)_L \times U(1)_Y$ is the gauge symmetry group of the Standard Model, B is the baryon and L is the lepton numbers, respectively. It was shown that Family Replicated Gauge Group model of the latter type fits the Standard Model fermion masses and mixing angles and describes all neutrino experiment data order magnitudewise using only 5 free parameters – five vacuum expectation values of the Higgs fields which break the Family Replicated Gauge Group symmetry to the Standard Model. The possibility of $[SU(5)]^3$ or $[SO(10)]^3$ unification at the GUT-scale $\sim 10^{18}$ GeV also is briefly considered.

1 Introduction

Trying to gain insight into Nature and considering the physical processes at very small distances, physicists have made attempts to explain the well-known laws of low-energy physics as a consequence of the more fundamental laws of Nature. The contemporary low-energy physics of the electroweak and strong interactions is described by the Standard Model (SM) which unifies the Glashow-Salam-Weinberg electroweak theory with QCD – the theory of strong interactions.

The gauge symmetry group in the SM is :

$$SMG = SU(3)_c \times SU(2)_L \times U(1)_Y \,, \tag{1}$$

which describes elementary particle physics up to the scale ≈ 100 GeV.

Recently it was shown that the Family Replicated Gauge Groups (FRGG) of the type $SU(n)^N \times SU(m)^N$ provide new directions for research in high energy physics and quantum field theory. In the Deconstruction of space-time models [1], the authors tried to construct renormalizable asymptotically free 4-dimensional gauge theories which dynamically generate a fifth dimension (it is possible to obtain more dimensions in this way). Such theories naturally lead to electroweak symmetry breaking, relying neither on supersymmetry nor on strong dynamics at the TeV scale. The new TeV physics is perturbative and radiative corrections to the Higgs mass are finite. Thus, we see that the family replicated gauge groups provide a new way to stabilize the Higgs mass in the Standard Model.

But there exists quite different way to employ the FRGG.

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2 Family Replicated Gauge Group as an extension of the Standard Model

The extension of the Standard Model with the Family Replicated Gauge Group :

$$G = (SMG)^{N_{fam}} = [SU(3)_c]^{N_{fam}} \times [SU(2)_L]^{N_{fam}} \times [U(1)_Y]^{N_{fam}} \tag{2}$$

was first suggested in the paper [2] and developed in the book [3] (see also the review [4]). Here N_{fam} designates the number of quark and lepton families. If $N_{fam} = 3$ (as our theory predicts and experiment confirms), then the fundamental gauge group G is:

$$G = (SMG)^3 = SMG_{1st\ fam.} \times SMG_{2nd\ fam.} \times SMG_{3rd\ fam.}, \tag{3}$$

or

$$G = (SMG)^3 = [SU(3)_c]^3 \times [SU(2)_L]^3 \times [U(1)_Y]^3. \tag{4}$$

The generalized fundamental group:

$$G_f = (SMG)^3 \times U(1)_f \tag{5}$$

was suggested by fitting the SM charged fermion masses and mixing angles in paper [5]. A new generalization of our FRGG-model was suggested in papers [6], where:

$$\begin{aligned} G_{\text{ext}} &= (SMG \times U(1)_{B-L})^3 \\ &\equiv [SU(3)_c]^3 \times [SU(2)_L]^3 \times [U(1)_Y]^3 \times [U(1)_{(B-L)}]^3 \end{aligned} \tag{6}$$

is the fundamental gauge group, which takes right-handed neutrinos and the see-saw mechanism into account. This extended model can describe all modern neutrino experiments, giving a reasonable fit to all the quark-lepton masses and mixing angles.

The gauge group $G = G_{ext}$ contains: $3\times8 = 24$ gluons, $3\times3 = 9$ W-bosons, and $3\times1+3\times1 = 6$ Abelian gauge bosons.

The gauge group $G_{\text{ext}} = (SMG \times U(1)_{B-L})^3$ undergoes spontaneous breakdown (at some orders of magnitude below the Planck scale) to the Standard Model Group SMG which is the diagonal subgroup of the non-Abelian sector of the group G_{ext}. As was shown in Ref. [7], 6 different Higgs fields: ω, ρ, W, T, ϕ_{WS}, ϕ_{B-L} break our FRGG-model to the SM. The field ϕ_{WS} corresponds to the Weinberg-Salam Higgs field of Electroweak theory. Its vacuum expectation value (VEV) is fixed by the Fermi constant: $\langle\phi_{WS}\rangle = 246$ GeV, so that we have only 5 free parameters – five VEVs: $\langle\omega\rangle$, $\langle\rho\rangle$, $\langle W\rangle$, $\langle T\rangle$, $\langle\phi_{B-L}\rangle$ to fit the experiment in the framework of the SM. These five adjustable parameters were used with the aim of finding the best fit to experimental data for all fermion masses and mixing angles in the SM, and also to explain the neutrino oscillation experiments.

Experimental results on solar neutrino and atmospheric neutrino oscillations from Sudbury Neutrino Observatory (SNO Collaboration) and the Super-Kamiokande Collaboration have been used to extract the following parameters:

$$\Delta m^2_{\text{solar}} = m_2^2 - m_1^2, \quad \Delta m^2_{\text{atm}} = m_3^2 - m_2^2, \quad \tan^2\theta_{\text{solar}} = \tan^2\theta_{12}, \quad \tan^2\theta_{\text{atm}} = \tan^2\theta_{23} \tag{7}$$

where m_1, m_2, m_3 are the hierarchical left-handed neutrino effective masses for the three families. We also use the CHOOZ reactor results. It is assumed that the fundamental Yukawa couplings in our model are of order unity and so we make order of magnitude predictions. The typical fit is shown in Table I. As we can see, the 5 parameter order of magnitude fit is encouraging.

Table 1. Best fit to conventional experimental data. All masses are running masses at 1 GeV except the top quark mass which is the pole mass.

	Fitted	Experimental
m_u	4.4 MeV	4 MeV
m_d	4.3 MeV	9 MeV
m_e	1.6 MeV	0.5 MeV
m_c	0.64 GeV	1.4 GeV
m_s	295 MeV	200 MeV
m_μ	111 MeV	105 MeV
M_t	202 GeV	180 GeV
m_b	5.7 GeV	6.3 GeV
m_τ	1.46 GeV	1.78 GeV
V_{us}	0.11	0.22
V_{cb}	0.026	0.041
V_{ub}	0.0027	0.0035
$\Delta m^2_\odot$	9.0×10^{-5} eV^2	5.0×10^{-5} eV^2
$\Delta m^2_{\rm atm}$	1.7×10^{-3} eV^2	2.5×10^{-3} eV^2
$\tan^2\theta_\odot$	0.26	0.34
$\tan^2\theta_{\rm atm}$	0.65	1.0
$\tan^2\theta_{\rm chooz}$	2.9×10^{-2}	$< 2.6 \times 10^{-2}$

There are also 3 see-saw heavy neutrinos in this model (one right–handed neutrino in each family) with masses: M_1, M_2, M_3. The model predicts the following neutrino masses:

$$m_1 \approx 1.4 \times 10^{-3}\ \text{eV}, \quad m_2 \approx 9.6 \times 10^{-3}\ \text{eV}, \quad m_3 \approx 4.2 \times 10^{-2}\ \text{eV} \tag{8}$$

– for left-handed neutrinos, and

$$M_1 \approx 1.0 \times 10^{6}\ \text{GeV}, \quad M_2 \approx 6.1 \times 10^{9}\ \text{GeV}, \quad M_3 \approx 7.8 \times 10^{9}\ \text{GeV} \tag{9}$$

– for right-handed (heavy) neutrinos.

Finally, we conclude that our theory with the FRGG-symmetry is very successful in describing experiment. The best fit gave the following values for the VEVs:

$$\langle W \rangle \approx 0.157, \quad \langle T \rangle \approx 0.077, \quad \langle \omega \rangle \approx 0.244, \quad \langle \rho \rangle \approx 0.265 \tag{10}$$

in the "fundamental units", $M_{Pl} = 1$, and

$$\langle \phi_{B-L} \rangle \approx 5.25 \times 10^{15}\ \text{GeV} \tag{11}$$

which gives the see-saw scale: the scale of breakdown of the $U(1)_{B-L}$ groups ($\sim 5 \times 10^{15}$ GeV).

3 The Problem of Monopoles in the Standard and Family Replicated Models

The aim of the present Section is to show, following Ref. [8], that monopoles cannot be seen in the Standard Model and in its usual extensions in the literature up to the Planck scale:

$M_{Pl} = 1.22 \times 10^{19}$ GeV, because they have a huge magnetic charge and are completely confined or screened. Supersymmetry does not help to see monopoles.

In theories with the FRGG-symmetry the charge of monopoles is essentially diminished. Then monopoles can appear near the Planck scale and change the evolution of the fine structure constants $\alpha_i(t)$ (here $i = 1, 2, 3$ corresponds to $U(1)$, $SU(2)$ and $SU(3)$), $t = \log(\mu^2/\mu_R^2)$, where μ is the energy variable and μ_R is the renormalisation point.

Let us consider the "electric" and "magnetic" fine structure constants:

$$\alpha = \frac{g^2}{4\pi} \quad \text{and} \quad \tilde{\alpha} = \frac{\tilde{g}^2}{4\pi}, \tag{12}$$

where g is the coupling constant, and $\tilde{g}$ is the dual coupling constant (in QED: $g = e$ (electric charge), and $\tilde{g} = m$ (magnetic charge)).

The Renormalization Group Equation (RGE) for monopoles is:

$$\frac{d(\log \tilde{\alpha}(t))}{dt} = \beta(\tilde{\alpha}). \tag{13}$$

With the scalar monopole beta-function we have:

$$\beta(\tilde{\alpha}) = \frac{\tilde{\alpha}}{12\pi} + (\frac{\tilde{\alpha}}{4\pi})^2 + \cdots = \frac{\tilde{\alpha}}{12\pi}(1 + 3\frac{\tilde{\alpha}}{4\pi} + \cdots). \tag{14}$$

The last equation shows that the theory of monopoles cannot be considered perturbatively at least for

$$\tilde{\alpha} > \frac{4\pi}{3} \approx 4. \tag{15}$$

And this limit is smaller for non-Abelian monopoles.

Let us consider now the evolution of the SM running fine structure constants. The usual definition of the SM coupling constants is given in *the Modified minimal subtraction scheme* ($\overline{\mathrm{MS}}$):

$$\alpha_1 = \frac{5}{3}\alpha_Y, \quad \alpha_Y = \frac{\alpha}{\cos^2\theta_{\overline{\mathrm{MS}}}}, \quad \alpha_2 = \frac{\alpha}{\sin^2\theta_{\overline{\mathrm{MS}}}}, \quad \alpha_3 \equiv \alpha_s = \frac{g_s^2}{4\pi}, \tag{16}$$

where α and α_s are the electromagnetic and $SU(3)$ fine structure constants respectively, Y is the weak hypercharge, and $\theta_{\overline{\mathrm{MS}}}$ is the Weinberg weak angle in $\overline{\mathrm{MS}}$ scheme. Using RGEs with experimentally established parameters, it is possible to extrapolate the experimental values of the three inverse running constants $\alpha_i^{-1}(\mu)$ from the Electroweak scale to the Planck scale (see Fig. 1).

In this connection, it is very attractive to include gravity. The quantity:

$$\alpha_g = \left(\frac{\mu}{\mu_{Pl}}\right)^2 \tag{17}$$

plays the role of the running "gravitational fine structure constant" (see Ref. [8]) and the evolution of its inverse is presented in Fig. 1 together with the evolutions of $\alpha_i^{-1}(\mu)$.

Assuming the existence of the Dirac relation: $g\tilde{g} = 2\pi$ for minimal charges, we have the following expression for the renormalised charges g and $\tilde{g}$ [9]:

$$\alpha(t)\tilde{\alpha}(t) = \frac{1}{4}. \tag{18}$$

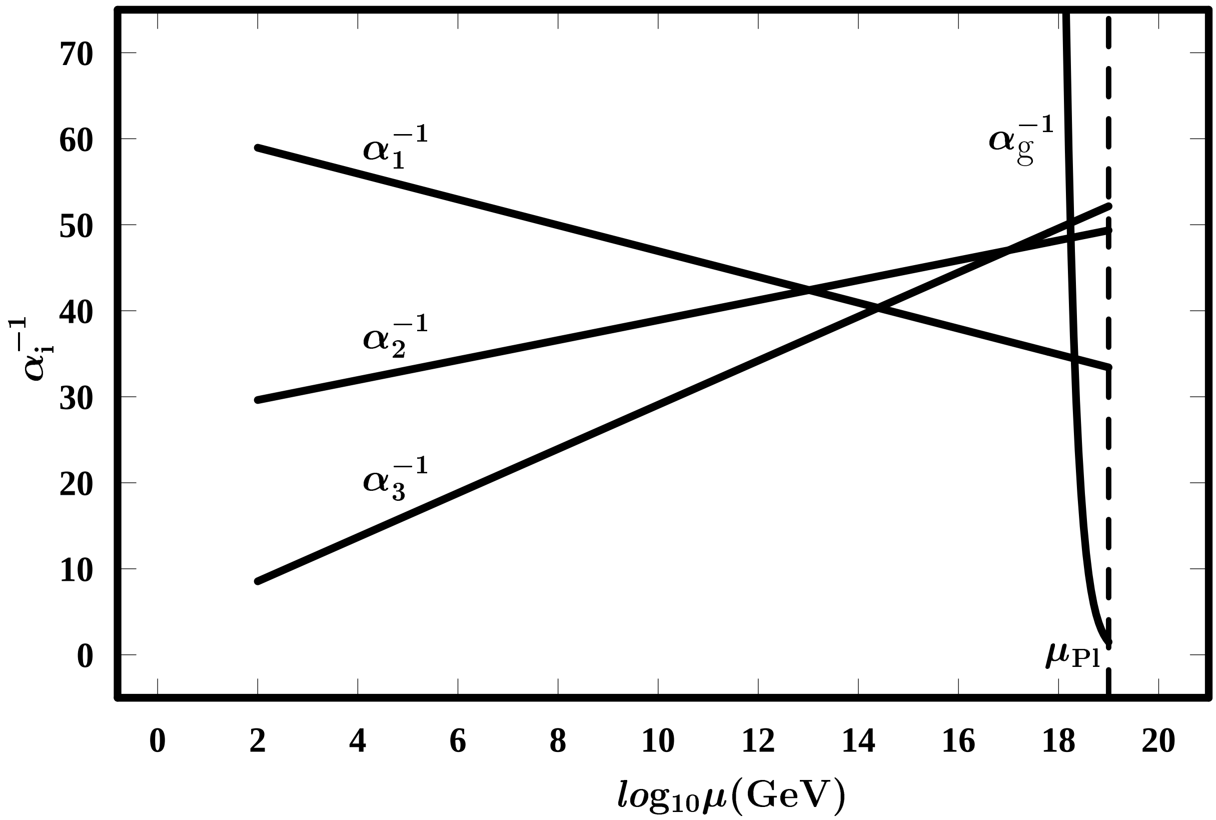

Figure 1.

Using the Dirac relation, it is easy to estimate (in the simple SM) the Planck scale value of $\tilde{\alpha}(\mu_{Pl})$ (minimal for $U(1)_Y$ gauge group):

$$\tilde{\alpha}(\mu_{Pl}) = \frac{5}{3}\alpha_1^{-1}(\mu_{Pl})/4 \approx 55.5/4 \approx 14\,. \tag{19}$$

This value is really very big compared with the estimate (15) and, of course, with the critical coupling $\tilde{\alpha}_{crit} \approx 1$, corresponding to the confinement – deconfinement phase transition in the lattice $U(1)$ gauge theory. Clearly we cannot make a perturbation approximation with such a strong coupling $\tilde{\alpha}$. It is hard for such monopoles not to be confined.

There is an interesting way out of this problem if one wants to have the existence of monopoles, namely to extend the SM gauge group so cleverly that certain selected linear combinations of charges get bigger electric couplings than the corresponding SM couplings. That could make the monopoles which, for these certain linear combinations of charges, couple more weakly and thus have a better chance of being allowed "to exist".

An example of such an extension of the SM that can impose the possibility of allowing the existence of free monopoles is just Family Replicated Gauge Group Model (FRGGM).

FRGGs of type $[SU(N)]^{N_{fam}}$ lead to the lowering of the magnetic charge of the monopole belonging to one family:

$$\tilde{\alpha}_{one\ family} = \frac{\tilde{\alpha}}{N_{fam}}. \tag{20}$$

For $N_{fam} = 3$, for $[SU(2)]^3$ and $[SU(3)]^3$, we have: $\tilde{\alpha}^{(2,3)}_{one\ family} = \tilde{\alpha}^{(2,3)}/3$. For the family replicated group $[U(1)]^{N_{fam}}$ we obtain:

$$\tilde{\alpha}_{one\ family} = \frac{\tilde{\alpha}}{N^*} \tag{21}$$

where $N^* = \frac{1}{2}N_{fam}(N_{fam}+1)$. For $N_{fam} = 3$ and $[U(1)]^3$, we have: $\tilde{\alpha}^{(1)}_{\text{one family}} = \tilde{\alpha}^{(1)}/6$ (six times smaller!). This result was obtained previously in Ref. [10].

According to the FRGGM, at some point $\mu = \mu_G < \mu_{Pl}$ (or really in a couple of steps) the fundamental group $G \equiv G_{\text{ext}}$ undergoes spontaneous breakdown to its diagonal subgroup:

$$G \longrightarrow G_{diag.subgr.} = \{g, g, g || g \in SMG\}, \tag{22}$$

which is identified with the usual (low-energy) group SMG.

In the Anti-GUT-model [2, 3] the FRGG breakdown was considered at $\mu_G \sim 10^{18}$ GeV. But the aim of this investigation is to show that we can see quite different consequences of the extension of the SM to FRGGM, if the G-group undergoes the breakdown to its diagonal subgroup (that is, SMG) not at $\mu_G \sim 10^{18}$ GeV, but at $\mu_G \sim 10^{14}$ or 10^{15} GeV, *i.e.* before the intersection of $\alpha_2^{-1}(\mu)$ with $\alpha_3^{-1}(\mu)$ at $\mu \approx 10^{16}$ GeV. In this case, in the region $\mu_G < \mu < \mu_{Pl}$ there are three $SMG \times U(1)_{B-L}$ groups for the three FRGG families, and we have a lot of fermions, mass protected or not mass protected, belonging to usual families or to mirror ones. In the FRGGM the additional 5 Higgs bosons, with their large VEVs, are responsible for the mass protection of a lot of new fermions appearing in the region $\mu > \mu_G$. Here we denote the total number of fermions N_F, which is different to N_{fam}.

Also the role of monopoles can be important in the vicinity of the Planck scale: they give contributions to the beta-functions and change the evolution of the $\alpha^{-1}(\mu)$. Finally, we obtain the following RGEs:

$$\frac{d(\alpha_i^{-1}(\mu))}{dt} = \frac{b_i}{4\pi} + \frac{N_M^{(i)}}{\alpha_i}\beta^{(m)}(\tilde{\alpha}_{U(1)}) \tag{23}$$

where b_i are given by the following values:

$$\begin{aligned} b_i &= (b_1, b_2, b_3) \\ &= (-\frac{4N_F}{3} - \frac{1}{10}N_S, \quad \frac{22}{3}N_V - \frac{4N_F}{3} - \frac{1}{6}N_S, \quad 11N_V - \frac{4N_F}{3}). \end{aligned} \tag{24}$$

The integers N_F, N_S, N_V, N_M are respectively the total numbers of fermions, Higgs bosons, vector gauge fields and scalar monopoles in the FRGGM considered in our theory. In our FRGG model we have $N_V = 3$, because we have 3 times more gauge fields ($N_{fam} = 3$), in comparison with the SM and one Higgs scalar monopole in each family.

We have obtained the evolutions of $\alpha_i^{-1}(\mu)$ near the Planck scale by numerical calculations for: $\mu_G = 10^{14}$ GeV, $N_F = 18$, $N_S = 6$, $N_M^{(1)} = 6$, $N_M^{(2,3)} = 3$. Fig. 2 shows the existence of the unification point.

We see that in the region $\mu > \mu_G$ a lot of new fermions, and a number of monopoles near the Planck scale, change the one-loop approximation behaviour of $\alpha_i^{-1}(\mu)$ which we had in the SM. In the vicinity of the Planck scale these evolutions begin to decrease, as the Planck scale $\mu = \mu_{Pl}$ is approached, implying the suppression of asymptotic freedom in the non-Abelian theories. Fig. 2 gives the following Planck scale values for the α_i:

$$\alpha_1^{-1}(\mu_{Pl}) \approx 13 \quad \alpha_2^{-1}(\mu_{Pl}) \approx 19 \quad \alpha_3^{-1}(\mu_{Pl}) \approx 24\,. \tag{25}$$

Fig. 2 demonstrates the unification of all gauge interactions, including gravity (the intersection of α_g^{-1} with α_i^{-1}), at

$$\alpha_{GUT}^{-1} \approx 27 \quad \text{and} \quad x_{GUT} \approx 18.4\,. \tag{26}$$

Here we can expect the existence of $[SU(5)]^3$ or $[SO(10)]^3$ (SUSY or not SUSY) unification.

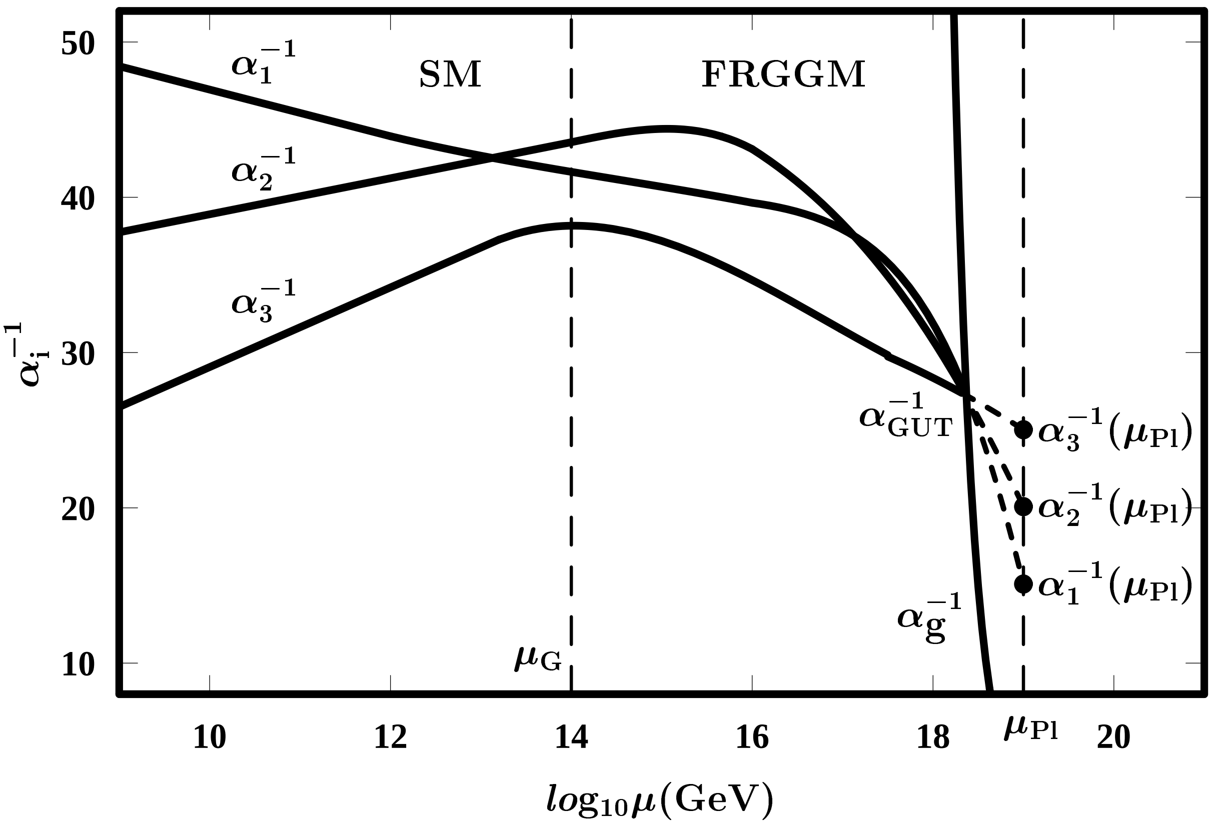

Figure 2.

Considering the predictions of such a theory for low-energy physics and cosmology, maybe in future we shall be able to answer the question: Does the unification $[SU(5)]^3$ or $[SO(10)]^3$ really exist near the Planck scale?

Recently F. S. Ling and P. Ramond [11] considered the group of symmetry $[SO(10)]^3$ and showed that it explains the observed hierarchies of fermion masses and mixings.

Acknowledgements

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[1] N. Arkani-Hamed, A. G. Cohen and H. Georgi, Phys. Rev. Lett. **86** (2001) 4757; Phys. Lett. B **516** (2001) 395.

[2] D. L. Bennett, H. B. Nielsen and I. Picek, Phys. Lett. B **208** (1988) 275.

[3] C. D. Froggatt and H. B. Nielsen, *Origin of Symmetries*, Singapore, World Scientific, 1991.

[4] L. V. Laperashvili, Phys. Atom. Nucl. **57** (1994) 471 [Yad. Fiz. **57** (1994) 501].

[5] C. D. Froggatt, M. Gibson, H. B. Nielsen and D. J. Smith, Int. J. Mod. Phys. A **13** (1998) 5037.

[6] H. B. Nielsen and Y. Takanishi, Nucl. Phys. B **604** (2001) 405; Phys. Lett. B **507** (2001) 241; C. D. Froggatt, H. B. Nielsen and Y. Takanishi, Nucl. Phys. B **631** (2002) 285.

[7] H. B. Nielsen and Y. Takanishi, Phys. Lett. B **543** (2002) 249.

[8] L. V. Laperashvili, D. A. Ryzhikh and H. B. Nielsen, Int. J. Mod. Phys. A, in press [arXiv:hep-th/0211224]; arXiv:hep-th/0212213.

[9] L. V. Laperashvili and H. B. Nielsen, Mod. Phys. Lett. A **14** (1999) 2797.

[10] D. L. Bennett and H. B. Nielsen, Int. J. Mod. Phys. A **9** (1994) 5155.

[11] F. S. Ling and P. Ramond, Phys. Rev. D **67** (2003) 115010.